Lecture 3 Interest Rates and Bond Analysis

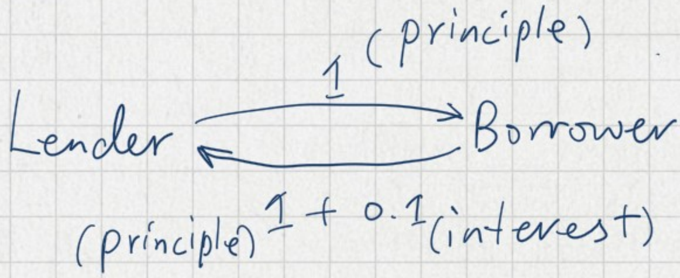
(2019. 2. 23)

- **Recap**
  - Finance : resource allocation { in time / under uncertainty (risk) }
  - Asset pricing : $X \Rightarrow P$

    or equivalently $X \Rightarrow E(\tilde{r})$

    (forecasting of X is a much broader topic than FE)
  - Main topics in this course
    - equilibrium pricing : Supply and Demand $\Rightarrow P$
    - No-Arbitrage pricing : some $P \Rightarrow$ other P
    - Financial frictions

3.1 Interest Rates in the Real World



Interest : a sum of money

Interest rate : a ratio (= interest / principle)

credit spread = risky (interest) rate − riskfree (interest) rate

3.2 Simple interest vs. Compound interest

- Simple interest

$$A(1+nr)$$

A: principle; n: n years; r: interest rate (annual rate)

- Compound interest (interest on interest)

$A(1+r)^n$ compound once per year

$A\left(1+\frac{r}{m}\right)^{mn}$ compound m times per year

Example: "1 year interest rate is 10%"

simple: $100 \times (1+10\%) = 110$

compound 2 times per year:

$100 \times (1+5\%)^2 = 110.25$

- Continuous compounding (compound at every moment)

$m \to +\infty$

$$\lim_{m\to\infty} A\left(1+\frac{r}{m}\right)^{mn} = A \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{\frac{m}{r}\cdot nr} = Ae^{nr}$$

$$\left(\because e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right)$$

- "Rule of 72"

with interest rate r, how many years will it take to double the principle?

$= 72/r$

To double China's GDP in 2020 compared with 2010

average GDP growth in these 10 years should be

$7.2\% = 7.2 \div 10 \div 100$

$(1+r)^t = 2 \Rightarrow t\ln(1+r) = \ln 2$

$\Rightarrow t \cdot r \approx \ln 2 \quad (\because \ln(1+r) \approx r)$

$\Rightarrow t \approx \ln 2 / r \approx \frac{0.693}{r}$

Rule of 69, 70, 69.3 and <u>72</u> (can be divided by 1, 2, 3, 4, 6, 8, 12)

3.3 Financial decision

- future value (FV)   present value (PV)

$$FV = PV(1+r)^n \qquad FV = PV \cdot e^{rn}$$

$$PV = \frac{FV}{(1+r)^n} \qquad PV = FV \cdot e^{-rn}$$

discount, discount rate

- Net Present Value (NPV)

time	0	1	2	3
cash flow	−100	30	60	40

$r = 10\%$

$$NPV = -100 + \frac{30}{1+0.1} + \frac{60}{(1+0.1)^2} + \frac{40}{(1+0.1)^3} = 6.9$$

(Do it)

if $r = 20\%$

$$NPV = -100 + \frac{30}{1+0.2} + \frac{60}{(1+0.2)^2} + \frac{40}{(1+0.2)^3} = -10.2$$

(DON'T do)

- Internal Rate of Return (IRR)

Rate that makes $NPV = 0$.

$$0 = -100 + \frac{30}{1+IRR} + \frac{60}{(1+IRR)^2} + \frac{40}{(1+IRR)^3}$$

$\Rightarrow IRR = 13.7\%$

NOTE: IRR is ONLY determined by the cash flow of the project (not affected by market interest rate r)!

$IRR > r$ (Do it)

$IRR < r$ (DON'T do)

- Example : IRR of JD Baitiao

Don't be fooled by 0.5% service fee rate, interest rate is 10% (annual rate)

计算京东白条买iPhoneX的利息率（2019年2月23日数据）

年息	**0.102**
月息	0.0085 <<<改变这个格子里的数值 来让折现值与当前售价相等

计算商品分期的折现值

贴现月数	每期分期	贴现因子	现值
1	1081.14	1.0085	1072.03
2	1081.14	1.017072	1062.99
3	1081.14	1.025717	1054.03
4	1081.14	1.034436	1045.15
5	1081.14	1.043229	1036.34
6	1081.14	1.052096	1027.61
总计			6298.15

商品当前售价

6299

iPhoneX 6299.00

6 periods 1081.14

2019年2月23日京东白条购买iPhoneX（未登录账号）

- Reinvestment risk

$$100 = \frac{30}{1+IRR} + \frac{60}{(1+IRR)^2} + \frac{40}{(1+IRR)^3}$$

$$\Rightarrow 100 \times (1+IRR)^3 = 30 \times (1+IRR)^2 + 60(1+IRR) + 40$$

In order to get $100 \times (1+IRR)^3$ in the end, cash flows generated in the middle should be able to earn IRR (Assumption)

Reinvestment risk : cannot earn IRR

Reinvestment risk can be ignored for industrial investment projects (because we can always reinvest cash into the project), but NOT for bond investment.

3.4 Value of Bonds

Face value 面值	Maturity/期限	Coupon 票息	Current Price 价格
A: 100 (principle)	1	0	95
B: 100	2	5	97

	0	1	2
A:	−95	100	
B:	−97	5	100+5

- coupon rate 票息率 $= \frac{\text{coupon}}{\text{Face}}$
- yield to maturity (YTM) = IRR of a bond

到期收益率 (interest rate of a bond)

B: $97 = \frac{5}{1+y} + \frac{100+5}{(1+y)^2} \Rightarrow y = 6.65\%$

(Don't forget reinvestment risk!)

QUESTION: Is YTM affected by market interest rate?

YES, because bond price is affected by market rate.

- Spot rate (zero rate) = IRR of a zero coupon bond

即期利率 ← discount rate used to calculate PV.

$r_1 = \frac{100}{95} - 1 = 5.26\%$ (Bond A)

$97 = \frac{5}{1+5.26\%} + \frac{105}{(1+r_2)^2} \Rightarrow r_2 = 6.69\%$ (Bond B)

(bootstrap method)

	Face	maturity	coupon	price
C :	100	2	6	?

$$P_C = \frac{6}{1+r_1} + \frac{106}{(1+r_2)^2} = 98.83$$

yield curve (curve of YTM)

- Forward rate (远期利率)

$$(1+r_1)(1+fr) = (1+r_2)^2$$

$$(1+5.26\%)(1+fr) = (1+6.69\%)^2 \Rightarrow fr = 8.13\%$$

The market forecasts 1-year rate to rise to 8.13% from 5.26%.

Forward rate: expected future interest rate

- Summary

How to compare different bonds?

Bond price (X), coupon rate (X), YTM(IRR) (√)

How to price a new bond? (How to discount cashflow?)

Spot rate (bootstrap method)

How to estimate market expectation?

Forward rate

3.4.4 Duration

y – ytm

$$P = \sum_{i=1}^{n} c_i e^{-y t_i} \quad (c_i \text{ – cashflow at } t_i)$$

- $$D = \frac{\sum_{i=1}^{n} t_i c_i e^{-y t_i}}{P} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-y t_i}}{P}\right]$$

average waiting time for a bond.

$$dP = \sum_{i=1}^{n} -t_i c_i e^{-y t_i} \, dy$$

$$= -P \frac{\sum_{i=1}^{n} t_i c_i e^{-y t_i}}{P} \, dy$$

$$= -P \cdot D \cdot dy$$

$$\Rightarrow \frac{\Delta P}{P} = -D \cdot \Delta y$$

The bigger the duration, the more sensitive of bond price to interest rate move. (yield curve parallel shift 平行移动)

- Duration strategy (久期策略)
  - expect $r\uparrow$ → smaller D.
  - expect $r\downarrow$ → bigger D.

- Immunization (久期免疫)

  D(assets) = D(Liabilities)

  immune to interest rate change