Lecture 7 Some Remarks on CAPM (2019.3.11)

Recap

CAPM (SML)

$$E(r_i) - r_f = \beta_i [E(r_M) - r_f], \quad \beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

SML applies to all assets (asset pricing equation)

CML only applies to efficient portfolios ($r_f + M$)

7.1 Risks from the Perspective of CAPM

- Mean vs. Variance : Tradeoff between return and risk.
- Not all volatilitis (variance) are risks, because some volatilities can be diversified away.
- Holding all risky assets (the entire market, M) is the ultimate diversification
- For any asset i,

  Part of σ_i uncorrelated to σ_M

  — diversifiable risk (idiosyncratic risk)

  NOT rewarded in asset prices

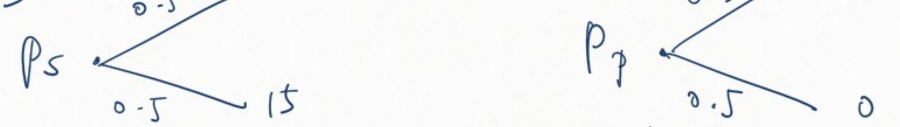
  Part of σ_i correlated to σ_M

  — undiversifiable risk (systematic risk)

  Rewarded in asset prices

Question 1 (from Box 1-3)

Steel | Pharmaceutical



Steel company has a smaller volatility, but a bigger β (as steel industry is more closely related to the macroeconomy and the market).

$\therefore P_s < P_p$

Question 2 : Is it possible that $E(r_i) < r_f$ for some i ?

Yes. $E(r_i) < r_f$ if $\beta_i < 0$

unemployment insurance

Question 3 : $E(r_i) = E(r_j)$, $\sigma_i < \sigma_j$

Should investors always choose i rather than j ?

Yes and No.

- Yes Part: $u(r) = E(r) - A\sigma^2(r)$

  Higher σ, lower utility.

- No Part: - At equilibrium, i and j are all holded by investors (as a part of M)

  - Might be $\beta_i > \beta_j$

- Reconciliation of Yes and No.

  Mean-Variance preference only applies to fully diversified portfolios, but is not applicable to individual assets.

7.3 Estimation of CAPM

Define excessive return (with a little bit abuse of notation)

$\tilde{r}_i \triangleq r_i - r_f$, $\tilde{r}_M \triangleq r_M - r_f$

SML $\Rightarrow$ $E(\tilde{r}_i) = \beta_i E(\tilde{r}_M)$

Econometric model $\tilde{r}_i = \alpha_i + \beta_i \tilde{r}_M + \tilde{\varepsilon}_i$

(Single Index Model)

OLS estimation $\hat{\beta}_i = \dfrac{\sigma_{iM}}{\sigma_M^2}$

By definition of OLS $cov(\beta_i \tilde{r}_M, \tilde{\varepsilon}_i) = 0$

$$Var(\tilde{r}_i) = Var(\alpha_i + \beta_i \tilde{r}_M + \tilde{\varepsilon}_i)$$
$$= Var(\beta_i \tilde{r}_M) + Var(\tilde{\varepsilon}_i) + 2cov(\beta_i \tilde{r}_M, \tilde{\varepsilon}_i)$$
$$= \underbrace{\beta^2 \sigma_M^2}_{\text{systematic risk}} + \underbrace{\sigma_{\varepsilon_i}^2}_{\text{idiosyncratic risk}}$$

7.3 Applications of CAPM

7.3.1 Determination of discount rate

Estimation of β for the investment project, $\beta \to r$.

Example: Gordon model

$g = 0.1$, $D_1 = 10$, $\beta = 1.5$, $r_f = 0.05$, $r_M - r_f = 0.1$

$$r = r_f + \beta(r_M - r_f) = 0.05 + 1.5 \times 0.1 = 0.2$$

$$S_0 = \frac{D_1}{r-g} = \frac{10}{0.2-0.1} = 100$$

7.3.2 Simplify portfolio optimization

Parameters need to be estimated in M-V analysis.

Mean | Variance (Variance-Covariance Matrix)

$$\bar{\mathbf{r}} = \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \vdots \\ \bar{r}_N \end{bmatrix} \qquad \mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$

For N assets, number of parameters $= N + \frac{N(N+1)}{2}$

3000 stocks, millions of parameters!

$$\sigma_{ij} = cov(\tilde{r}_i, \tilde{r}_j) = cov(\alpha_i + \beta_i \tilde{r}_M + \tilde{\varepsilon}_i, \alpha_j + \beta_j \tilde{r}_M + \tilde{\varepsilon}_j)$$
$$= \beta_i \beta_j cov(\tilde{r}_M, \tilde{r}_M) + cov(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j)$$
$$= \beta_i \beta_j \sigma_M^2$$

$$\Sigma = \sigma_M^2 \begin{bmatrix} \beta_1^2 & \beta_1\beta_2 & \cdots & \beta_1\beta_N \\ \beta_2\beta_1 & \beta_2^2 & \cdots & \beta_2\beta_N \\ \vdots & \vdots & \ddots & \vdots \\ \beta_N\beta_1 & \beta_N\beta_2 & \cdots & \beta_N^2 \end{bmatrix}$$

Number of parameters

$$\underset{(\text{mean})}{N} + \underset{(\beta\text{'s})}{N} + \underset{(\sigma_M^2)}{1}$$

7.3.3 Investment Performance Evaluation

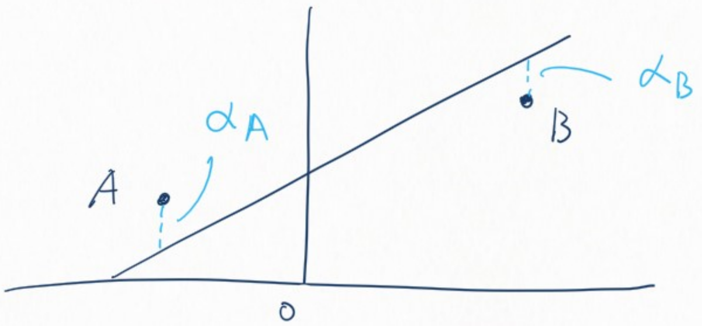
$$SR_i = \frac{\bar{r}_i - r_f}{\sigma_i}$$ Only applicable to fully diversified funds.

- For funds focusing on specific areas.

  Jensen's Alpha (Jensen Index)

  $$\text{Alpha} = (\bar{r}_i - r_f) - \beta_i(\bar{r}_M - r_f)$$

  Vertical distance to SML.



- A is doing a better job even if $\bar{r}_A < \bar{r}_B$ and $\sigma_A > \sigma_B$.
  (A question from BOX 1-3)
- By combining a fund with positive Alpha with M, one can beat M (obtaining higher SR).

- In equilibrium, all assets should have 0 Alpha. But in reality, the market is not always in equilibrium (although it is assumed to be so in economic theories). Some assets can have positive Alpha (Warren Buffett). By encorporating these assets into portfolios, old market portfolio is replaced by a new market portfolio — like the old equilibrium been replaced by a new equilibrium.
  — An issue to be revisited in Lecture 25 (Financial Arts)

7.3.4 Alpha-Beta Separation

$r_\alpha = r_f + 0.03 + 1.5(r_M - r_f) + \varepsilon$ 100 million

$r_H = -0.5 r_f + 1.5 r_M = r_f + 1.5(r_M - r_f)$

100 million (buy 150 million M with borrowed 50 million)

$r_p = r_\alpha - r_H$

$= 0.03 + \varepsilon$

If ε is small, 0.03 alpha is separated, and can be transferred to other assets.
— Portable Alpha

- Story of Bridgewater

7.5 Limitation of CAPM

Partial Equilibrium → General Equilibrium (C-CAPM)
Static Model → Dynamic Model (I-CAPM)
Single Index → Multiple index (APT);

BOX: Remarks on Bill Gate's Diversification

Story: As the founder of Microsoft, Bill owned about 45% of Microsoft stocks.

Bill hired portfolio mangers to manage his wealth, and his wealth has been gradually diversified away from Microsoft stocks.

In nowadays, Bill only holds about 1% of Microsoft stocks. By diversification, Bill's total wealth has been increasing steadily in past decades, and has reached about 90 billion USD.

But if Bill continued to hold 45% of Microsoft stocks, his total wealth would be about 400 billion USD in nowadays!

Question: Does Bill's story suggest that diversification is WRONG?

Answer: Investment decision

 $\longrightarrow E(\underset{\sim}{X}) \longrightarrow$ Investment

diversification applies to this phase

What is wrong (if there is something wrong) in Bill's story is that investors (including Bill himself) failed to recognize the potential Microsoft has. But it is unfair to ask investors to have such vision (think about Nokia, Motorola).