

# Final Review<sup>\*</sup> †

Yumin Hu‡

PKU

June 1, 2019

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<sup>\*</sup>This is a review lecture on financial economics 2019S taught by Xu Gao.

<sup>†</sup>Wish you all the best for the final exam.

<sup>‡</sup>Contact email: hhym110730@pku.edu.cn

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- ① Introduction
- ② Absolute pricing
- ③ Relative pricing
- ④ Financial fractions
- ⑤ Conclusion

## Overview

这份讲义的 25 讲可以大致分成五部分。第一部分包含第 1 到第 4 讲，是课程的介绍部分，意在让那些初次接触金融学的读者了解金融的基本概念。第二部分包含第 5 讲到第 12 讲，是均衡资产定价的部分，介绍了均值方差分析、CAPM、C-CAPM 等内容。第三部分包含第 13 讲到第 19 讲，是无套利定价的部分，介绍了风险中性定价、二叉树、对冲等内容。第四部分包括第 20 讲到第 24 讲，重点在于把信息不对称、有限套利、非理性等摩擦因素引入金融分析，以丰富金融理论对现实世界的解释力。第 25 讲自成一部分，站在金融理论的外部来看理论的方法论基础和应用边界。

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# Introduction Lecture 1-4

## ① Introduction to Financial Economics

- ▶ Finance: intertemporal and uncertainty
- ▶ Asset pricing
  - Equilibrium pricing
  - No arbitrage pricing
  - Rate of return: good assets V.S. bad assets
- ▶ Corporate finance: fractions, asymmetric information
- ▶ Behavioral finance → Effective market

## ② Bonds

- ▶ IRR & Reinvestment Risk
- ▶ Spot Rate( $r_i$ ) & Yield to maturity( $y_i$ ) & Forward Rate( $fr_{i,j}$ )
- ▶ Duration

## ③ Stocks

- ▶ DDM & Gordon model
- ▶ Transversality Condition(TVC)
- ▶ PE ratio
- ▶ Dividend decision & Fisher Separation Theorem

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# CAPM Lecture 5-7

## ① Preference: Mean-Variance Analysis

- ▶ ex ante & ex post
- ▶ Risk premium, Sharpe ratio

## ② Behavior and Equilibrium

CML(efficient frontier, Two-fund separation):  $E(r_i) - r_f = \frac{\sigma_i}{\sigma_M} [E(r_M) - r_f]$

SML(pricing model):  $E(r_i) - r_f = \beta_i [E(r_M) - r_f]$

## ③ Properties: CAPM

- ▶ Determination of discount rate & Portable alpha

## ④ Three Questions(7.1.2)

- ▶ Steel V.S. Pharmaceutical
- ▶ It is possible that  $E(r_i) < r_f$
- ▶  $E(r_i) = E(r_j), \sigma_i < \sigma_j$ , investors choose which one?

# C-CAPM Lecture 8-12

## ① Preference: Expected Utility(Lecture 8)

- ▶ Expected Utility Theorem: Rational+Continuous+Independence
- ▶ Risk aversion & Certainty equivalent
- ▶ Utility functions(HARA,CARA,CRRA)

## ② Behavior: Behavior under risks(Lecture 9)

- ▶ Risky Assets [Different State]
  - Proposition1:  $a^* > 0 \Leftrightarrow E(\tilde{r}) > r_f$
  - Proposition2:  $a''(w_0) > 0 \Leftrightarrow R'_A(\cdot) < 0$
  - Proposition3:  $e(w_0) > 1 \Leftrightarrow R'_R(\cdot) < 0$
- ▶ Savings under risk( $R_B$  is more risky than  $R_A$ ) [Different Time]
  - Proposition4:  $s_A < s_B \Leftrightarrow P_R(sR) > 2$
  - Precautionary saving V.S. Substitution effect

### ③ Equilibrium: General Equilibrium(Lecture 10-11)

- ▶ Property of best risk sharing

- ① Consumptions of all consumers are perfectly correlated
- ② Consumption is only determined by aggregated risk
- ③ Wilson Theorem:  $\frac{dc_{ks}}{de_s} = \frac{T_k(c_{ks})}{\sum_{k=1}^K T_k(c_{ks})}$

- ▶ Aggregated risk V.S. Idiosyncratic risk

- ▶ Representative consumer, HARA  $\frac{(c-d)^{1-\gamma}}{1-\gamma}$

### ④ Properties: C-CAPM(Lecture 12)

- ▶ SDF:  $\tilde{m} = \delta \frac{u'(\tilde{c}_1)}{u'(c_0)}$ ,  $p = E(\tilde{m}\tilde{x})$

- ▶ Risk-free rate:  $r_f \approx \frac{1-\delta}{\delta} + R_R \bar{g} - \frac{1}{2} R_R P_R \sigma_g^2$  (Determination)

- ▶ Risk premium:  $E[\tilde{r}_j] - r_f = -\frac{\delta(1+r_f)}{u'(c_0)} cov(u'(\tilde{c}_1), \tilde{r}_j)$  (Covariance)

### ⑤ Two puzzles

- ▶ Risk free rate puzzle

- ▶ Equity premium puzzle

- Two economic forces (time smoothing and state smoothing)
- One parameter

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# APT Lecture 13

## ① APT model [Exercise 13.1]

- CAPM → Fama-French three-factor model

$$\tilde{r}_i - r_f = \alpha_i + \beta_{i,M}(\tilde{r}_M - r_f) + \tilde{\epsilon}_i$$

$$\tilde{r}_i - r_f = \alpha_i + \beta_{i,M}(\tilde{r}_M - r_f) + \beta_{i,S}SMB + \beta_{i,H}HML + \tilde{\epsilon}_i$$

- $\tilde{r}_i = \alpha_i + \sum_{k=1}^K \beta_{i,k} \tilde{f}_k + \tilde{\epsilon}_i, \quad i = 1, 2, \dots, N$

$$\tilde{r}_p = \sum_{i=1}^N w_i \alpha_i + (\sum_{i=1}^N w_i \beta_{i,1}) \tilde{f}_1 + \dots + (\sum_{i=1}^N w_i \beta_{i,K}) \tilde{f}_K + \sum_{i=1}^N w_i \tilde{\epsilon}_i$$

$$\lambda_k = E(\tilde{r}_{pk}) - r_f - E(\tilde{f}_k)$$

$$E(\tilde{r}_i) = r_f + \sum_{k=1}^K \beta_{i,k} [\lambda_k + E(\tilde{f}_k)] = r_f + \sum_{k=1}^K \beta_{i,k} [E(\tilde{r}_{pk}) - r_f]$$

- ② Parameters:  $\tilde{f}_k \ \beta_{i,k} \ \tilde{\epsilon}_i \ r_{pk} \ \lambda_k$

- ③ Application: Portable alpha, Statistical arbitrage

# NA-Pricing Lecture 14-15

## ① Options & Futures [Exercise 14.1 14.2]

- ▶ Forward price v.s. Expectation of spot price in the future
- ▶ Put-call Parity(European v.s. American)
- ▶ Options and Complete: State-index asset, butterfly spread
- ▶ Pricing idea: replicate bond/options, Risk Neutral World

## ② Fundamental Theorem of Asset Pricing

- ▶ Complete N.A.  $\Leftrightarrow \exists !\varphi$  s.t.  $P_j = \sum_{s=1}^S \varphi_s x_s^j$  [Exercise 15.2]
  - $P = \sum_{s=1}^S \pi_s \frac{\varphi_s}{\pi_s} x_s = \sum_{s=1}^S \pi_s m_s x_s = E(\tilde{m}\tilde{x})$
  - $P = \sum_{s=1}^S \varphi_s x_s = e^{-r} \sum_{s=1}^S \frac{\varphi_s}{\sum_{k=1}^S \varphi_k} x_s^j = e^{-r} \sum_{s=1}^S q_s x_s = e^{-r} E^Q[\tilde{x}]$
  - $P = \sum_{s=1}^S \pi_s \frac{\delta u'(c_{1,s})}{u'(c_0)} x_s \rightarrow q_s = \delta \pi_s \frac{u'(c_{1,s})}{u'(c_0)} / \sum_{s=1}^S \delta \pi_s \frac{u'(c_{1,s})}{u'(c_0)} = \frac{\pi_s u'(c_{1,s})}{\sum_{s=1}^S \pi_s u'(c_{1,s})}$

- ▶ NA  $\Leftrightarrow$  Risk-neutral  $\Leftrightarrow$  Martingale

# Multiperiod pricing(Tree Model) Lecture 16

- ① Dynamic complete: Long-lived asset  $\geq$  Maximum of successor node
- ② Law of iterated expectation:  $E_t(\tilde{x}) = E_t[E_{t+1}(\tilde{x})]$
- ③ Dynamic pricing

- ▶ Martingale: Define  $\hat{S}_t = e^{-rt} S_t$  as deflated stock price, we have  
$$E_0[\hat{S}_2] = E_0[\hat{S}_1] = \hat{S}_0 \text{ & } E_1[\hat{S}_2] = \hat{S}_1$$
- ▶  $q = \frac{e^r - d}{u - d}$
- ▶  $C_u = e^{-r}[qC_{uu} + (1 - q)C_{ud}], C_d = e^{-r}[qC_{ud} + (1 - q)C_{dd}]$
- ▶  $C_0 = e^{-2r}[q^2 C_{uu} + 2q(1 - q)C_{ud} + (1 - q)^2 C_{dd}]$
- ▶ **Derivatives Payoff Function:** European options, American options, Floating strike lookback call options [**Exercise 16.1**], Asian options

# Optimal Stopping(Bellman Equation) Lecture 17

## ① Problem1

$$V(R, G) = \max\{0, \frac{R}{R+G}[1 + V(R-1, G)] + \frac{G}{G+R}[-1 + V(R, G-1)]\}$$

## ② Problem2

$$P = \max\{\max\{K - S, 0\}, \frac{1}{1+r}[qP_u + (1-q)P_d]\}$$

## ③ Problem3

$$V_s =$$

$$\min\{B_t, \frac{1}{1+r_s}[q(\bar{r}B_t + B_t - B_{t+1} + V_{su}) + (1-q)(\bar{r}B_t + B_t - B_{t+1} + V_{sd})]\}$$

[Exercise 17.2]

# BS Equation(Continuous time) Lecture 18

## ① Concepts

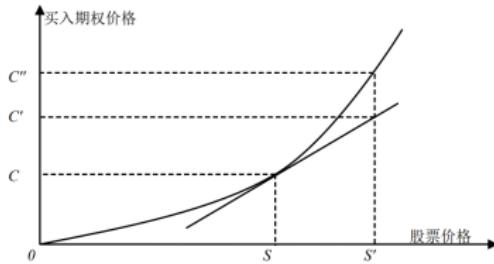
- ▶ Random Walk, Brownian motion, Wiener Process
- ▶ Ito's Lemma
- ▶ geometric Brownian motion(Assets price)

## ② BS equation(European Option)

- ▶  $C_0 = S_0 N(d_1) - e^{-rT} K N(d_2)$
- ▶  $P_0 = -S_0 N(-d_1) + e^{-rT} K N(-d_2)$
- ▶ Put-Call Parity(Verify)  $P_0 + S_0 = C_0 + K e^{-rT}$
- ▶ Intuition:  $S_0 N(d_1), N(d_2)$

# Dynamic Hedging Lecture 19

- ① Naked position & Covered position, Stop loss strategy
- ② Delta Hedge:  $\Delta = \frac{\partial \Pi}{\partial S}$
- ③ Greeks [Exercise 19.1]
  - ▶ Gamma:  $\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 \Pi}{\partial S^2}$ . Curvature & Hedging error
  - ▶ Vega:  $\nu = \frac{\partial \Pi}{\partial \sigma}$



- ④ Portfolio Insurance: replicate option [Exercise 19.2]
  - \*  $\Delta$  is the position, not flow

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# Asymmetric Information Lecture 20-21

## ① Moral Hazard(ex-post): Credit rationing

- ▶ IC constraint & IR constraint [**Exercise 20.1**]
- ▶ Application: Financial Accelerator, Debt Overhang, Debt-deflation, Fiscal & Monetary Policy

## ② Adverse Selection(ex-ante): Capital Structure

- ▶ MM Theory & Tradeoff Theory
- ▶ Pecking Order Theory
  - Information Intensity: Low → High
  - Internal Financing, External Financing(Debt, Equity)

# Maturity Mismatch Lecture 22

## ① DD Model [Exercise 22.2]

- ▶ Autarky(ATK)
- ▶ Market (MKT, Open in period 1)
- ▶ Central Planner(BST)
- ▶ Bank(BNK)

## ② Bank

- ▶ Maturity Transformation(Cash Pool)
- ▶ Bank Run(Self-fulfilling) & Deposit Insurance
- ▶ Morale Hazard & Regulation
- ▶ Shadow Banking
- ▶ Internet Finance [Exercise 22.1]

# Behavioral Finance Lecture 23

## ① Limits to Arbitrage

- ▶ Fundamental Risk & Implementation Costs & Noise Trader Risk

## ② Performance-based Arbitrage [Exercise 23.1]

- ▶ Performance-based → Expand market volatility

## ③ Systematic Bias

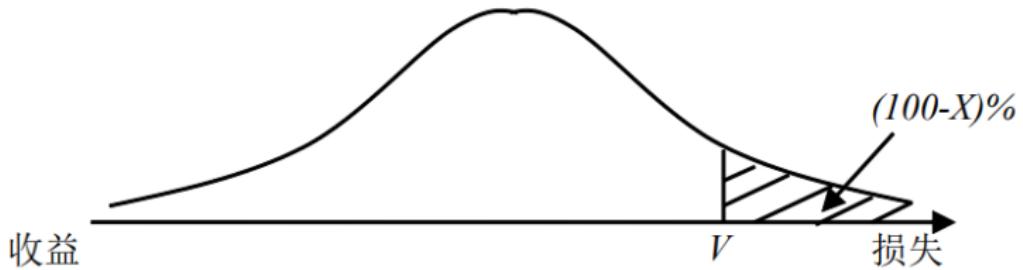
- ▶ Overconfidence & Optimism & Belief Perseverance
- ▶ Prospect Theory & Loss Aversion

## ④ Comments: Behavioral Finance

# Financial Risk Lecture 24

## ① Market Risk & Credit Risk & Operation Risk

- ▶ Greeks: Delta, Gamma, Vega, Theta, Rho
- ▶ Value at Risk:  $V(T,X)$



## ② Subprime Mortgage Crisis

- ▶ ABS & CDO
- ▶ CDS & Synthetic CDO

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# Conclusion

Schedule: <https://finaecon2019s.github.io/FinaEcon2019S/>

12	Mon,5.6	第20讲,道德风险与信贷配给 (slides)	提取码: gtny	HW11:20.1 Due:5月13日	从本讲开始, 我们进入金融摩擦的领域 理解本讲模型设定和信贷配给理论的四个应用
12	Sat,5.11	第21讲,逆向选择与资本结构 (slides)	提取码: 975L		理解模型的setup, 掌握信息强度与啄虚假说的概念
13	Mon,5.13	第22讲,银行与期限错配 (slides)	提取码: PYt8	HW12:22.2 Due:5月20日	理解模型setup; 理解银行实现的期限转换功能, 及其对应带来的期限错配问题
14	Mon,5.20	第23讲,行为金融学初探 (slides)	提取码: c6P1	HW13:23.1 Due:5月27日	理解模型setup, 有限套利
15	Mon,5.27	第24讲,风险管理与次贷危机 (slides)	提取码: o4Ly		掌握相关概念, 例如风险价值度、希腊字母、CDO、CDS、合成CDO等等, 次贷危机爆发的原因
18	Mon,6.17	期末考试			

## Grades

- ▶ <https://shimo.im/sheets/uc7QXLuatNwG8UBC/MODOC>
- ▶ 平时成绩  $23 + 2^*$ <sup>†</sup> & 期中考试 25 !!! DDL [23:00, June 16, 2019]
- ▶ 期末考试: 50 [2019 年 6 月 17 日 (周一) 18: 30-20: 30]

\*Method1: 教材勘误, PPT 勘误

†Method2: 教材答案征集 <https://www.wjx.cn/jq/11554255.aspx>

# Model

## NA Pricing

- ① APT [**Exercise 13.1**]
- ② Fundamental Theorem of Asset Pricing [**Exercise 15.2**]
- ③ Multiperiod Pricing [**Exercise 16.1**]
- ④ Optimal Stopping [**Exercise 17.2**]
- ⑤ Dynamic Hedging [**Exercise 19.1 & 19.2**]

## Financial Frictions

- ① Credit Rationing [**Exercise 20.1**] & Capital Structure
- ② Diamond-Dybvig Model [**Exercise 22.2**]
- ③ Performance-based Arbitrage [**Exercise 23.1**]

End

时间：2019.06.17， 18: 30-20: 30

地点：理教 302（111）理教 303（110）

50-60 计算 & 40-50 简答，请务必携带计算器

May you suffer the examination and be stronger

Financial Economics 2019 Spring, Xu Gao(徐高)

TA: 胡裕民<sup>1</sup>, 张子萱<sup>2</sup>

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<sup>1</sup>hhym110730@pku.edu.cn

<sup>2</sup>zhangzixuan19970429@outlook.com

